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Measurement of Fluid Properties for Magnetoplasmadynamic Power Generators

Eighth Quarterly Technical Summary Report

(1 February—30 April 1965)

Contract No. Nonr-4104(00) and Amendment 1

Order No.: ARPA 420 Project Code No. 3980

EDR 4290

Allison Division • General Motors

Indianapolis, Indiana

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EDR 4296

25 May 1965

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FOREWORD

This technical summary report by the Research Department of the Allison Division of General Motors Corporation. The work reported was accomplished under Contract Nonr-4104(00) and Amendment No. 1 thereto.

The program was sponsored by the Advanced Research and Project Agency through the Power Branch of the Office of Naval Research under the direction of ARPA and Mr. J. A. Satkowski of ONR.

The report was written by H. E. Wilhelm.

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I. INTRODUCTION

This report describes the progress achieved on Contract ONR Nonr-4104(00) from 1 February through 30 April 1965.

Experimental effort during the report period was directed toward completion of the new test section, magnet, and heater. In the theoretical portion of the program, Ohm's law was derived for nonisothermal plasma with thermal diffusion. This derivation is presented herein.

II. EXPERIMENTAL INVESTIGATIONS

The magnet was operated with the test section in place. The highest magnetic field obtainable in the steady-state condition for extended time periods was 20,000 gauss. A calibration curve was established showing magnet current versus magnetic field. The estimated error was less than 5%.

Vacuum checking of the completed system was accomplished. A vacuum of 100 microns can be maintained with a mechanical pump when pumping the entire system. The system is completed and the first helium checkout runs are scheduled for the near future.

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III. THEORETICAL INVESTIGATIONS

OHM'S LAW FOR NONISOTHERMAL PLASMA WITH THERMAL DIFFUSION

In a plasma, two types of thermal diffusion forces can be distinguished[1] *—those conditioned by conduction heat currents and those conditioned by barodiffusion heat currents (radiation heat currents are disregarded).[2] Thermal diffusion forces are of importance for these processes which are based on momentum transfer between plasma components. In this contribution, the transport of electrical current in nonisothermal plasmas is treated for an arbitrary degree of ionization. It is shown, among other things, that the thermal barodiffusion effect (TBDE) changes quantitatively the electrical conductivity parallel and transverse to the magnetic field and the Hall coefficient. Concerning the kinetic theory of the TBDE, it is demonstrated that inclusion of the effect of intercomponent momentum transfer on the relaxation of the individual heat currents in the components is indispensable.

The following considerations apply strictly only to infrasonic barodiffusion velocities:

$$\frac{1}{2}\mu_{sr}\,(\vec{v}_s - \vec{v}_r)^2 \ll kT_{sr}$$

This condition is satisfied for most applications— $\mu_{sr} = (m_s\, m_r)/(m_s + m_r)$ = reduced mass and $T_{sr} = \mu_{sr}\left[(T_s/m_s) + (T_r/m_r)\right]$ = reduced temperature of the s- and r-components.

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*Superscripts refer to references in Section IV.

THERMAL DIFFUSION EFFECT AND ELECTRICAL CONDUCTION

Consider a plasma composed of electrons (e), single charged ions (i), and neutrals (a) containing an electromagnetic field $\vec{E}$, $\vec{B}$. The components s = e, i, a are separately assumed to be in approximate local statistical equilibrium,[3] but relative to each other in thermal nonequilibrium, $T_s \neq T_r$. The condition for the latter is $E_{eff} \gtrsim E_P$, where E_{eff} is the effective electrical field acting on the electron component and E_P is the critical plasma field.[4] Under these conditions the transport of linear momentum of the s-component, interacting via friction forces $\sim(\vec{v}_r - \vec{v}_s)$ and thermal diffusion forces $\sim [(\vec{q}_r/n_r m_r) - (\vec{q}_s/n_s m_s)]$ with the remaining components r, is described by:[5,6,7]

$$n_s m_s \left[\frac{\partial}{\partial t} + \vec{v}_s \cdot \nabla\right]\vec{v}_s + \nabla \cdot \vec{P}_s - n_s e_s (\vec{E} + \vec{v}_s \times \vec{B}) = - n_s m_s \sum_{r \neq s} \tau_{sr}^{-1} \left[(\vec{v}_s - \vec{v}_r) + \alpha_{sr} \frac{\mu_{sr}}{kT_{sr}} \left(\frac{\vec{q}_s}{n_s m_s} - \frac{\vec{q}_r}{n_r m_r}\right)\right] \tag{1}$$

The designations are standard. The possible combinations of interactions between the charged and neutral particles in plasma are treated as Coulomb interactions, C.I. [Debye - radius $D = \left(\frac{4}{k}\sum_s \frac{n_s e_s^2}{T_s}\right)^{-1/2}$ as maximum impact parameter; transport cross section $Q_{sr} = \frac{\pi}{2}\left(\frac{e_s e_r}{k T_{sr}}\right)^2 \cdot \ln \Lambda_{sr}$, $\Lambda_{sr} = D\,(|e_s e_r|/3k\,T_{sr})^{-1}$][8] and elastic sphere interactions, S.I. [interaction radii r_s and r_r; transport cross section $Q_{sr} = \pi(r_s + r_r)^2$].[9] The coefficients are taken throughout from the 13-moment approximation.[7,10] According to the latter, the coefficients of the thermal diffusion forces are

$$\alpha_{sr} = -3/5 \quad \ldots\text{C.I.}; \quad \alpha_{sr} = +1/5 \quad \ldots\text{S.I.} \tag{2}$$

The characteristic reciprocal times determining the relaxation of the velocity fields are:

$$\tau_{sr}^{-1} = \frac{8}{3}\sqrt{\frac{2\,kT_{sr}}{\pi\,\mu_{sr}}}\,\frac{\mu_{sr}}{m_s}\, n_r Q_{sr} \quad \ldots\text{C.I., S.I.} \tag{3}$$

If radiation heat currents are negligible, the heat flux in the s-component is determined by the mechanisms of conduction $\sim\nabla T_s$ and barodiffusion (Dufour effect) $\sim(\vec{v}_s - \vec{v}_r)$:

$$\vec{q}_s = -\lambda_s \boldsymbol{\Omega}_s \left[\nabla T_s + \sum_{r \neq s} \tau_{sr}^{-1} \epsilon_{sr} \frac{\mu_{sr}}{k} (\vec{v}_s - \vec{v}_r)\right],$$

$$\lambda_s = \frac{5}{2} \frac{k}{m_s} p_s \tau_s \tag{4}$$

m,

The influence of the magnetic field on the heat conduction process is represented by the operator ($\vec{B}^o = \vec{B} / B$):

$$\boldsymbol{\Omega}_s = \Omega_s^{\circ} + \Omega_s^{\parallel} \vec{B}^o (\vec{B}^o \cdot\ ) - \Omega_s^{\perp} (\vec{B}^o \times\ ),$$

$$\Omega_s^{\circ} = (1 + \omega_s^2 \tau_s^2)^{-1}, \Omega_s^{\parallel} = \omega_s^2 \tau_s^2 \Omega_s^{\circ}, \Omega_s^{\perp} = \omega_s \tau_s \Omega_s^{\circ} \tag{5}$$

Note that the gyration frequency is defined as dependent on the sign of the particle, $e_s : \omega_s = e_s B/m_s$. The coefficients of the Dufour effect are:

$$\epsilon_{sr} = \begin{cases} -\frac{3}{5}\left[3 \frac{T_s}{T_{sr}} - 2 + \frac{4}{3 \ln \Lambda_{sr}} \left(1 - \frac{T_{sr}}{T_s}\right)\right] & \text{. . . C. I.,} \\ +\frac{1}{5}\left[16 \frac{T_{sr}}{T_s} + 3 \frac{T_s}{T_{sr}} - 18\right] & \text{. . . . S. I.} \end{cases} \tag{6}$$

Evaluation of the relaxation times, τ_s, for the heat fluxes in the components according to the 13-moment method yields, considering intercomponent as well as viscous momentum transfer:*

$$\begin{aligned} \tau_a^{-1} &= \frac{4}{5} \tau_{aa}^{-1} + \frac{19}{8} \tau_{ai}^{-1} + 3\, \tau_{ae}^{-1} + \left[A_a\right], \\ \tau_e^{-1} &= \frac{4}{5} \tau_{ee}^{-1} + \frac{13}{10} \tau_{ea}^{-1} + \frac{13}{10} \tau_{ei}^{-1} + \left[A_e\right], \\ \tau_i^{-1} &= \frac{4}{5} \tau_{ii}^{-1} + \frac{19}{8} \tau_{ia}^{-1} + 3\, \tau_{ie}^{-1} + \left[A_i\right], \end{aligned} \tag{7}$$

where τ_{ss}^{-1} and τ_{sr}^{-1} are given in Equation (3). The expressions A_a, A_e, and A_i stand for terms

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*In Equation (7), quantitatively unimportant factors, $f_{ss} = 1 - (1 - \Lambda_{ss}^{-2})(2 \ln \Lambda_{ss})^{-1} \simeq 1$ arising from the Coulomb interactions, are omitted for reasons of simplicity.

associated with higher approximations not considered here.[11] Physically, the appearance of the cross terms τ_{sr}^{-1} ($s \neq r$) follows from the interrelation of conductive and barodiffusion heat fluxes, Equation (4).

In contrast to the relations established in Equation (7), it was assumed in Reference 7 that only collisions between like particles are relevant for the considered relaxation ($\tau_s^{-1} \approx \frac{4}{5} \tau_{ss}^{-1}$). This approximation, however, is insufficient as is proved later.

Assuming the Debye-radius, D, to be small compared to the dimension of the plasma system, to which the results shall apply, space charge effects can be neglected throughout, $n_e = n_i$, and an ionization degree defined by $\kappa = n_e/(n_e + n_a) = n_i/(n_i + n_a)$. Equation (1), taken for $s = e$ and $s = i$, yields two independent equations. From these, one readily obtains after elimination of $\vec{v}_e$ and $\vec{v}_i$ by means of the relations:*

$$\vec{v}_e = \vec{v} + (\vec{v}_e - \vec{v}_i) + (1 - \kappa)(\vec{v}_i - \vec{v}_a) \tag{8}$$

and

$$\vec{v}_i = \vec{v} + (1 - \kappa)(\vec{v}_i - \vec{v}_a) \tag{9}$$

which follow immediately from the definition for the plasma velocity, $\vec{v} = \sum n_s m_s \vec{v}_s / \sum n_s m_s$, and of $(\vec{v}_e - \vec{v}_a)$ by means of

$$0 \equiv (\vec{v}_e - \vec{v}_i) - (\vec{v}_e - \vec{v}_a) + (\vec{v}_i - \vec{v}_a) \tag{10}$$

two equations for the conduction current density, $\vec{j}$, and ion slip current density, $\vec{I}$, defined by:

$$\vec{j} \equiv n_e e_e (\vec{v}_e - \vec{v}_i) \text{ and } \vec{I} = n_i e_i (\vec{v}_i - \vec{v}_a) \tag{11}$$

These equations are:

$$\left[- Q_e \vec{j} + R_e (\vec{j} \cdot \vec{B}^o) \vec{B}^o + S_e \vec{j} \times \vec{B}^o\right] + \sigma_e \left[\vec{E} + \vec{v} \times \vec{B} - \vec{E}_e^d\right] = + \frac{\tau_{ei}}{\tau_{ei} + \tau_{ea}} \left[- K_e \vec{I} + L_e (\vec{I} \cdot \vec{B}^o) \vec{B}^o + M_e \vec{I} \times \vec{B}^o\right] \tag{12}$$

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*The corresponding equations used by Cowling[12] are applicable only for small ionization degrees, $\kappa \ll 1$.

and

$$\left[- Q_i \vec{j} + R_i (\vec{j} \cdot \vec{B}^o) \vec{B}^o + S_i \vec{j} \times \vec{B}^o\right] + \sigma_i \left[\vec{E} + \vec{v} \times \vec{B} - \vec{E}_i^d\right] = - \frac{\tau_{ie}}{\tau_{ia}} \left[- K_i \vec{I} + L_i (\vec{I} \cdot \vec{B}^o) \vec{B}^o + M_i \vec{I} \times \vec{B}^o\right] \tag{13}$$

In Equations (12) and (13), the diffusion fields, caused by the inhomogeneities in the plasma components, are in nonviscous approximation $\left[\nabla \cdot \overleftrightarrow{P}_s \cong \nabla p_s\right.$, and because of $m_e << m_{i, a}$, $\left. d\vec{v}_s/dt \cong (- \nabla p + \vec{j} \times \vec{B})/\Sigma n_s m_s\right]$ given by ($s = e$, i):

$$\vec{E}_s^d = - \frac{\left[\kappa \frac{m_s}{m_i} \nabla p - \nabla p_s\right]}{n_s e_s} - \frac{m_s}{e_s} \sum_{r \neq s} \tau_{sr}^{-1} \alpha_{sr} \frac{\mu_{sr}}{kT_{sr}} \left[\frac{\lambda_s \Omega_s \nabla T_s}{m_s} \quad \frac{\lambda_r \Omega_r \nabla T_r}{n_r m_r}\right] \tag{14}$$

Further, σ_e and σ_i designate the common conductivity coefficients of the partially and fully ionized plasma, respectively,

$$\sigma_e = n_e e_e^2 / m_e (\tau_{ei}^{-1} + \tau_{ea}^{-1}) \tag{15}$$

$$\sigma_i = n_i e_i^2 / m_i \tau_{ie}^{-1} \tag{16}$$

The expressions K_s, ... S_s, ($s = e$, i) are given by, if one makes the convention that for $s = e$, $\nu \equiv i$ and for $s = i$, $\nu \equiv e$:

$$\begin{aligned} Q_s &= 1 - (a_s \Omega_s^o + b_s \Omega^o_{\nu \neq s} + e_s) & K_s &= 1 - (d_s \Omega_s^o - h_s \Omega^o_{\nu \neq s} + f_s) \\ R_s &= a_s \Omega_s^{\parallel} + b_s \Omega_{\nu \neq s}^{\parallel} & L_s &= d_s \Omega_s^{\parallel} - h_s \Omega_{\nu \neq s}^{\parallel} \\ & & M_s &= d_s \Omega_s^{\perp} - h_s \Omega_{\nu \neq s}^{\perp} + (1-\kappa) \omega_s \tau_{sa} \\ S_e &= a_e \Omega_e^{\perp} + b_e \Omega_i^{\perp} + \omega_e (\tau_{ei}^{-1} + \tau_{ea}^{-1})^{-1} & S_i &= a_i \Omega_i^{\perp} + b_i \Omega_e^{\perp} - \kappa \omega_i \tau_{ie} \end{aligned} \tag{17}$$

where [for an application, note that some of the coefficients in Equations (18) and (19) are small of order $(m_e/m_{i,a})^1$ and $(m_e/m_{i,a})^2$]

$$
\begin{aligned}
a_e &= \frac{5}{2}\frac{T_e}{T_{ei}}\,\alpha_{ei}\,\epsilon_{ei}\,\frac{\tau_e(\tau_{ei}^{-1}+\tau_{ea}^{-1})^{-1}}{\tau_{ei}^2}\left[1+\frac{T_{ei}}{T_{ea}}\frac{\alpha_{ea}}{\alpha_{ei}}\frac{\tau_{ei}}{\tau_{ea}}\right]^{-1}\cdot\left[1+\frac{\epsilon_{ea}}{\epsilon_{ei}}\frac{\tau_{ei}}{\tau_{ea}}\right]^{-1} \\
b_e &= \left(\frac{m_e}{m_i}\right)^2\cdot\frac{5}{2}\frac{T_i}{T_{ei}}\,\alpha_{ei}\,\epsilon_{ie}\,\frac{\tau_i(\tau_{ei}^{-1}+\tau_{ea}^{-1})}{\tau_{ei}\,\tau_{ie}} \\
c_e &= +\left(\frac{m_e}{m_a}\right)^2\cdot\frac{5}{2}\frac{T_a}{T_{ea}}\,\alpha_{ea}\,\epsilon_{ae}\,\frac{\tau_a(\tau_{ei}^{-1}+\tau_{ea}^{-1})^{-1}}{\tau_{ea}\,\tau_{ae}} \\
d_e &= \frac{5}{2}\frac{T_e}{T_{ei}}\,\alpha_{ei}\,\epsilon_{ea}\,\frac{\tau_e}{\tau_{ei}}\left[1+\frac{T_{ei}}{T_{ea}}\frac{\alpha_{ea}}{\alpha_{ei}}\frac{\tau_{ei}}{\tau_{ea}}\right] \\
h_e &= \left(\frac{m_e}{2m_i}\right)\cdot\frac{5}{2}\frac{T_i}{T_{ei}}\,\alpha_{ei}\,\epsilon_{ia}\,\frac{\tau_i\,\tau_{ea}}{\tau_{ei}\,\tau_{ia}} \\
f_e &= \left(\frac{m_e}{2m_a}\right)\cdot\frac{5}{2}\frac{T_a}{T_{ea}}\,\alpha_{ea}\,\epsilon_{ai}\,\frac{\tau_a}{\tau_{ai}}\left[1+\frac{2m_e}{m_a}\frac{\epsilon_{ae}}{\epsilon_{ai}}\frac{\tau_{ai}}{\tau_{ae}}\right]
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
a_i &= \left(\frac{m_e}{m_i}\right)^2\cdot\frac{5}{2}\frac{T_i}{T_{ie}}\,\alpha_{ie}\,\epsilon_{ie}\,\frac{\tau_i}{\tau_{ie}}\left[1+\frac{m_i}{2m_e}\frac{T_{ie}}{T_{ia}}\frac{\alpha_{ia}}{\alpha_{ie}}\frac{\tau_{ie}}{\tau_{ia}}\right] \\
b_i &= \frac{5}{2}\frac{T_e}{T_{ie}}\,\alpha_{ie}\,\epsilon_{ei}\,\frac{\tau_e}{\tau_{ei}}\left[1+\frac{\epsilon_{ea}}{\epsilon_{ei}}\frac{\tau_{ei}}{\tau_{ea}}\right] \\
c_i &= -\left(\frac{m_e}{2m_a}\right)\cdot\frac{5}{2}\frac{T_a}{T_{ia}}\,\alpha_{ia}\,\epsilon_{ae}\,\frac{\tau_a\,\tau_{ie}}{\tau_{ia}\,\tau_{ae}} \\
d_i &= \left(\frac{m_e}{2m_i}\right)\cdot\frac{5}{2}\frac{T_i}{T_{ie}}\,\alpha_{ie}\,\epsilon_{ia}\,\frac{\tau_i}{\tau_{ie}}\left[1+\frac{m_i}{2m_e}\frac{T_{ie}}{T_{ia}}\frac{\alpha_{ia}}{\alpha_{ie}}\frac{\tau_{ie}}{\tau_{ia}}\right] \\
h_i &= \frac{5}{2}\frac{T_e}{T_{ie}}\,\alpha_{ie}\,\epsilon_{ea}\,\frac{\tau_e\,\tau_{ia}}{\tau_{ie}\,\tau_{ea}} \\
f_i &= \left(\frac{1}{4}\right)\cdot\frac{5}{2}\frac{T_a}{T_{ia}}\,\alpha_{ia}\,\epsilon_{ai}\,\frac{\tau_a}{\tau_{a_i}}\left[1+\frac{2m_e}{m_a}\frac{\epsilon_{ae}}{\epsilon_{ai}}\frac{\tau_{ai}}{\tau_{ae}}\right]
\end{aligned}
\tag{19}
$$

Equations (12) and (13) are readily solvable for the ion slip density, $\vec{I}$. Elimination of the latter and introduction of the operators (s = e, i),

$$\boldsymbol{\Sigma}_s \equiv K_s + U_s \vec{B}^o (\vec{B}^o \cdot) - M_s (\vec{B}^o \times) \tag{20}$$

and the abbreviations (s = e, i)

$$U_s = (K_s L_s + M_s^2) / (K_s - L_s) \tag{21}$$

$$\gamma = \frac{m_e}{m_i} \; \frac{\tau_{ia}}{\tau_{ei} + \tau_{ea}} \; \frac{K_e^2 + M_e^2}{K_i^2 + M_i^2} \tag{22}$$

lead to a generalized Ohm's law in the form:

Partially Ionized Plasma, $0<K \leq 1$:

$$\begin{aligned} &\left[Q_e (K_e + U_e) + \gamma Q_i (K_i + U_i) \vec{j}\right] \\ &- \left[R_e (K_e + U_e) + \gamma R_i (K_i + U_i)\right] (\vec{j} \cdot \vec{B}^o) \vec{B}^o \\ &- \left[(K_e S_e - Q_e M_e) + \gamma (K_i S_i - Q_i M_i)\right] \vec{j} \times \vec{B}^o \\ &- \left[(M_e S_e - Q_e U_e) + \gamma (M_i S_i - Q_i U_i)\right] (\vec{j} \times \vec{B}^o) \times \vec{B}^o \\ &= \sigma_e \boldsymbol{\Sigma}_e \left[\vec{E} + \vec{v} \times \vec{B} - \vec{E}_e^d\right] + J\, i\, \sigma_i \boldsymbol{\Sigma}_i \left[\vec{E} + \vec{v} \times \vec{B} - \vec{E}_i^d\right] \end{aligned} \tag{23}$$

Equation (23) indicates that the TBDE modifies, besides the operators, $\boldsymbol{\Sigma}_s$, all current terms, in particular those assoicated with the Hall effect[12] [of importance at free electron spiraling—i.e., $|\omega_e \, \omega|(\tau_{ei}^{-1} + \tau_{ea}^{-1})^{-1} \gtrapprox 1$] and the ion-slip effect[12] [of importance at free electron and ion spiraling—i.e., $|\omega_e/(\tau_{ei}^{-1} + \tau_{ea}^{-1})^{-1} \cdot \omega_i \tau_{ia} \gtrapprox 1$], while the term $\sim (\vec{j} \cdot \vec{B}^o) \vec{B}^o$ is due alone to thermal barodiffusion. For explanation, it is referred to the structure of the operator, $\boldsymbol{\Sigma}_s$, determing the heat fluxes and, thus, the thermal diffusion forces in magnetic field. The various expressions $\sim\gamma$ are not negligible for strong nonisothermal plasmas, $T_{es} \gg T_{sr}$, $(s, r \neq e)$, [in Equation (22), $\tau_{ia}/\tau_{ea} = \sqrt{(m_i/2\, m_e)\,(T_{ea}/T_{ia})}\; Q_{ea}/Q_{ia}$ and $\tau_{ia}/\tau_{ie} = \sqrt{(m_i/2m_e)\,(T_{ei}/T_{ia})}\;(n_i\, Q_{ei})/(n_a\, Q_{ia})$] especially if the ionization is appreciable (ion slip effect).

In most experimental situations the inhomogeneities in the components are sufficiently small to allow regarding the diffusion fields, $\vec{E}^d_s$, as unessential compared to the applied fields, $\vec{E}$, and induced fields $\vec{v} \times \vec{B}$. In this quasihomogeneous approximation, $\vec{E}^d_s \cong 0$, one can deduce from Equation (23) simple relations for the current density parallel ($\parallel$) and transverse ($\perp$) to the magnetic field by splitting the field vectors in $\parallel$ and $\perp$ components, $\vec{j}_\parallel = (\vec{j} \cdot \vec{B}^o)\, \vec{B}^o$, $\vec{j}_\perp = \vec{j} - j_\parallel = \vec{B}^o \times (\vec{j} \times \vec{B}^o)$, etc. Thus, one finds for the current transport along the $\vec{B}$ field lines

$$\vec{j}_\parallel = \sigma_\parallel \vec{E}_\parallel \tag{24}$$

where the "parallel conductivity" is given by [see also Equations (21) and (22)]:

$$\sigma_\parallel = \frac{\sigma_e (K_e + U_e) + \gamma \sigma_i (K_i + U_i)}{(Q_e - R_e)(K_e + U_e) + \gamma (Q_i - R_i)(K_i + U_i)} \tag{25}$$

Because $Q_s - R_s = Q_s\ (B = 0)$ and $K_s + U_s = (K_s^2 + M_s^2)/K_s\ (B = 0)$,

$$\sigma_\parallel = \frac{\sigma_e f}{Q_e\,(B{=}0)}, \quad f \equiv \left[1 + \frac{m_e}{m_i}\frac{\tau_{ia}}{\tau_{ea}}\frac{K_e\ (B{=}0)}{K_i\ (B{=}0)}\right]\cdot\left[1 + \frac{m_e}{m_i}\left(\frac{\tau_{ia}}{\tau_{ei} + \tau_{ea}}\right)\frac{K_e\ (B{=}0)}{K_i\ (B{=}0)}\frac{Q_i\ (B{=}0)}{Q_e\ (B = 0)}\right]^{-1} \tag{26}$$

Consequently, $\sigma_\parallel$ is independent of the magnetic field. Under conditions where $f \cong 1$, it is $\sigma > \sigma_e$, as $0 < Q_e\ (B{=}0) = 1 - (a_e + b_e + c_e) < 1$. Note, however, that for strong nonisothermy, $T_{es} \gg T_{sr}$ (s, r $\neq$ e) and appreciable ionization, $f \lesssim 1$. In the same way, one finds for the current transport transverse to the $\vec{B}$ field lines:

$$\vec{j}_\perp = \sigma_\perp \left[(\vec{E} + \vec{v} \times \vec{B}) - \beta\, (\vec{E} + \vec{v} \times \vec{B}) \times \vec{B}^o\right] \tag{27}$$

where the transverse conductivity is given by see also Equations (21) and (22) :

$$\sigma_\perp = \frac{(\sigma_e K_e + \gamma \sigma_i K_i)\, X - (\sigma_e M_e + \gamma \sigma_i M_i)\, Y}{X^2 + Y^2} \tag{28}$$

with

$$\begin{aligned} X &\equiv (K_e Q_e + M_e S_e) + \gamma (K_i Q_i + M_i S_i) \\ Y &\equiv (K_e S_e - M_e Q_e) + \gamma (K_i S_i - M_i Q_i) \end{aligned} \tag{29}$$

The quantity β has the meaning of an effective Hall coefficient given by

$$\beta = -\frac{(\sigma_e' K_e + \gamma \sigma_i K_i) Y + (\sigma_e M_e + \gamma \sigma_i M_i) X}{(\sigma_e K_e + \gamma \sigma_i K_i) X - (\sigma_e M_e + \gamma \sigma_i M_i) Y} > 0 \tag{30}$$

The complexity of the expressions for σ and β is caused primarily by the γ terms discussed previously. Suppressing the latter results in:

$$\sigma_\perp' = (\sigma_e/Q_e)(1 + \beta'^2)^{-1}, \ \beta' = -S_e/Q_e.$$

Suppressing in these relations the TBDE—$Q_e \to 1$, $K_e \to 1$, $S_e \to \omega_e (\tau_{ei}^{-1} + \tau_{ea}^{-1})^{-1}$—leads to the expressions of the elementary theory:

$$\sigma_\perp'' = \sigma_e (1 + \beta''^2)^{-1}, \ \beta'' = |\omega_e| (\tau_{ei}^{-1} + \tau_{ea}^{-1})^{-1}.$$

Comparison indicates that $\sigma_\perp' > \sigma_\perp''$ and $\beta' > \beta''$ for small magnetic fields with $\Omega_e^\circ \cong 1$, while for large magnetic fields with $\Omega_e^\circ \ll 1$, $\sigma_\perp' \cong \sigma_\perp''$ and $\beta' \cong \beta''$ (reduction of the transverse TBDE by strong magnetic fields).

From the selected collisional interactions, the elastic sphere model is much less favorable for the TBDE (e.g., it can be shown that for predominating electron neutral interaction, the TBDE affects the conductivity by less than 7.7%) than the Coulomb interactions. For a more quantitative illustration, the electrical conduction law, Equation (23), is now discussed for the important case of a plasma with Coulomb interactions $\left[K_s = 1,\ L_s = M_s = U_s = 0,\ \gamma = 0\right]$:

Fully Ionized Plasma, $\kappa = 1$:

$$\begin{aligned} \vec{j} - \frac{R_e^*}{Q_e^*}(\vec{j} \cdot \vec{B}^o)\vec{B}^o - \frac{S_e^*}{Q_e}\vec{j} \times \vec{B}^o \\ = \frac{\sigma_e^*}{Q_e^*}\left[\vec{E} + \vec{v} \times \vec{B} - \vec{E}_e^d\right] \end{aligned} \tag{31}$$

The coefficients σ_e^*, Q_e^*, R_e^*, S_e^* and the diffusion field, $\vec{E}_e^{d*}$, are obtained from the corresponding expressions without asterisk by setting in them terms $\sim \tau_{sa}^{-1}$ and τ_{as}^{-1}—interactions with neutrals a—identical with zero. Equation (31) exhibits, besides the conductive thermal diffusion effect, the TBDE and the Hall effect, where because of $\Sigma_e^* = 1$, a scalar, the effective electrical field acts directly on the plasma. It follows for the current transport along the $\vec{B}$ field lines:

$$\vec{j} = \sigma_{\parallel}^{*} \left[\vec{E} - \vec{E}_e^{d*}\right] \tag{32}$$

where the parallel conductivity is independent of the magnetic field and given by:

$$\sigma_{\parallel}^{*} = \frac{\sigma_e^{*}}{Q_e^{*} - R_e^{*}} = \frac{\sigma_e^{*}}{Q_e^{*}\ (B=0)} \tag{33}$$

Under consideration of Equations (7) and (17) and (18), one has explicitly

$$\left[b_e^{*} \sim \left(\frac{m_e}{m_i}\right)^2 << 1,\ T_{ei} \cong T_{ee} = T_e\right]:\ \sigma_{\parallel}^{*} \cong \sigma_e^{*}\ (13 + 4\sqrt{2})/4(1 + \sqrt{2}).$$

Accordingly, $\sigma_{\parallel}^{*} \cong 1.92\ \sigma_e^{*}$; this, about twice as high a conductivity value in comparison to σ_e^{*}, agrees well with that derived by Spitzer by taking into account the "electron-electron-interaction."[8] One finds for the current transport transverse to the $\vec{B}$ field lines:

$$\vec{j}_{\perp} = \sigma_{\perp}^{*} \left[(\vec{E} + \vec{v} \times \vec{B} - \vec{E}_e^{d})_{\perp} - \beta^{*}\ (\vec{E} + \vec{v} \times \vec{B} - \vec{E}_e^{d})_{\perp} \times \vec{B}^{o}\right] \tag{34}$$

The transverse conductivity and the effective Hall coefficient are given by:

$$\sigma_{\perp}^{*} = \frac{\sigma_e^{*}/Q_e^{*}}{1 + \beta^{*2}},\quad \beta^{*} = -\frac{S_e^{*}}{Q_e^{*}} > 0 \tag{35}$$

Under consideration of Equations (7) and (17) and (18), one has explicitly

$$\left[b_e^{*} \sim \left(\frac{m_e}{m_i}\right)^2 << 1,\ T_{ei} \cong T_{ee} = T_e\right]:\ \sigma_{\perp}^{*} = \sigma_e^{*} \left\{\left[1 - \frac{9}{13 + 4\sqrt{2}}\ (1 + \omega_e^2\ \tau_e^{*2})^{-1}\right]\right.$$

$$\left. \cdot\ (1 + \beta^{*2})\right\}^{-1},\ \text{and}\ \beta^{*} = +\left|\ \omega_e\right|\ \tau_{ei} \left[1 + \frac{90}{(13 + 4\sqrt{2})^2}\ (1 + \omega_e^2\ \tau_e^{*2})^{-1}\right]$$

$$\cdot \left[1 - \frac{9}{13 + 4\sqrt{2}}\ (1 + \omega_e^2\ \tau_e^{*2})^{-1}\right]^{-1}$$

It is seen that the TBDE produces an increase of the Hall coefficient. This effect is, however, the smaller, the larger the magnetic field intensity—e.g., $\beta^{*} \cong 2.52\ |\ \omega_e\ |\ \tau_{ei}$ for $\omega_e^2\ \tau_e^{*2} << 1$, while $\beta^{*} \cong |\ \omega_e\ |\ \tau_{ei}$ for $\omega_e^2\ \tau_e^{*2} >> 1$. The transverse conductivity is increased by the TBDE for small magnetic fields, $\omega_e^2\ \tau_e^{*2} << 1$, while for large magnetic fields, $\omega_e^2\ \tau_e^{*2} >> 1$, it has no influence—e.g., $\sigma_{\perp}^{*} \cong 1.92\ \sigma_e^{*}\ (1 + \omega_e^2\ \tau_{ei}^2)^{-1}$ for $\omega_e^2\ \tau_e^{*2} << 1$, while $\sigma_{\perp}^{*} \cong \sigma_e^{*}\ (1 + \omega_e^2\ \tau_{ei}^2)^{-1}$ for $\omega_e^2\ \tau_e^{*2} >> 1$.

FURTHER REMARKS

It is noticeable that by approximating the relaxation times of the heat fluxes, Equation (7), by the self-collision terms, $\tau_s = \frac{5}{4}\tau_{ss}$,* in the absence of strong magnetic fields, Q_e becomes smaller and smaller when approaching the region of predominating Coulomb interations—finally zero—and is negative throughout the region of full ionization. Completely analogous aspects are exhibited by Q_e when the electron density becomes smaller and smaller. As Q_e compares the opposing effects of intercomponent friction and thermal barodiffusion, this approximation would lead to physically unacceptable conductivity values ($\sigma \sim \frac{1}{Q_e}$!) and is, consequently, quantitatively and qualitatively unacceptable.

As the TBDE and the intercomponent friction effect are opposing each other and are for certain collision models of the same order of magnitude, their comparative consideration allows in a simple way checking the sufficiency or insufficiency of any kinetic approximation made. For example, such comparisons might be useful in validating the higher approximations of the Chapman-Enshog and 13-moment methods.

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*Corresponding to the first approximation of Chapman-Enshog.

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